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MODELING MAINTENANCE DATA BY EXPONENTIAL, WEIBULL, GAMMA, NORMA--ETC(U)
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The purpose of this analysis is to use maintenance repair time data collected from one-quarter truck in the exponential, Weibull, gamma, truncated normal, and lognormal distribution models to determine the best model for these data based on Messrs. Blanchard and Lowery's theory.

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CONTENT

CONTENTS (Continued)

MODELING MAINTENANCE DATA BY EXPONENTIAL, WEIBULL, GAMMA NORMAL AND LOGNORMAL DISTRIBUTIONS

1. INTRODUCTION

1.1 Objective

Messrs. Blanchard and Lowery's 1969 study, listed as reference number two, indicated that maintenance data model to the normal and lognormal distributions as opposed to some of the other frequently used distributions. This report evaluates a sample of Army vehicle maintenance data to determine how well it satisfies their conclusion.

1.2 Background

The time-to-repair data used in this analysis were derived from maintenance data which were taken on an Army M151A1 1/4 ton utility truck. These data were collected through the Army Integrated Equipment Record Maintenance Management System (TAERS). Several hundred time-to-repair unscheduled corrective actions were recorded from an Army vehicle. Of these, a random sample of 366 samples was utilized for this evaluation. Isolation, removal, replacement, alignment and verification times are not considered in this data sample analysis. The personnel responsible for restoring a piece of equipment to its original operable state were individuals with some formal training to be mechanics. History of these mechanics who record time-to-repair data indicated that they tended to round off their restoring times to the nearest one tenth hour. This will be discussed in more detail in a later section.

Maintainability time can be defined as "a characteristic of design and installation which is expressed as the ability to retain or restore an item to a specified condition within a given period of time." Ordinarily, these times are considered to be continuous random variables; however, these data appeared to be recorded by rounding off to the nearest one-tenth hour and possibly rounded to the nearest one-half hour. As a result, raw data in Table I (Page 28) based on random sample times will be analyzed using discrete random variables (x) since for each outcome (x_i), there is a $P(x_i) \geq 0$, $i = 1, 2 \ldots n$.

Figure I is a histogram of corrective repair times in hours based on the non-grouped data in Table II. Figure II is an identical histogram to Figure I except its interval widths were increased to adjust for round off in recording repair times. These round-off data correspond to adjusted intervals in Table III.

The fitting of these data to the widely used exponential, Weibull, gamma, normal, and lognormal distributions are analyzed in respective

sections of this report. Although, this report will show computations for both non-grouped data (Table II) and grouped data (Table III), the conclusion will be determined from adjusted interval data given in Table III. Each section utilizes the same basic interval widths, even though the forms of the exponential, Weibull, gamma, normal, and lognormal distributions are different.

A statistical comparison test using observed data with theoretical results of the assumed distributions had to be selected. Such test is called the "Goodness-of-fit test;" two are used in this analysis. These tests are the Chi-square (χ^2) and Kolmogorov-Smirnov (K-S) tests.

The χ^2 goodness-of-fit test can be used with large samples to test the validity of any assumed discrete or continuous distribution. The Kolmogorov-Smirnov is useful in large or small samples, however, it is restricted to testing the continuous distributions. Both goodness-of-fit tests were applied in this analysis, although the Kolmogorov-Smirnov (K-S) test analysis is not shown in this report. Figure III gives a pictorial representation of additional reasoning for not choosing the K-S Test. It is easily seen in this figure that if a curve were fitted through the center of maintenance points in the 0.5 interval, the standard deviation would be at least 0.2 as compared to a smaller number at a significant level of 0.10, 0.05, or 0.01. Consequently, the χ^2 goodness-of-fit test was decided to be the better of the two tests.

II. EXPONENTIAL DISTRIBUTION

Table I presents 366 individual corrective-maintenance repair times for all kinds of truck failures. Such data will be used in the assumed underlying distribution to compute an estimate of its parameter. The maximum likelihood method is used in the following equation to estimate the θ parameter of an exponential distribution $(1-\text{EXP}(-X/\theta))$

$$\hat{\theta} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{1}{\hat{\lambda}} = \text{Sample Mean}$$

$$\hat{\theta} = 242.8/366 = .6634$$

where

θ = Mean time to repair

N = Sample size

X = Maintenance repair time (Hours)

λ = Failure rate

With the estimated parameter, a statistical goodness-of-fit test is used to evaluate the assumed distribution. Such a goodness-of-fit test is the Chi-square (χ^2) test. Three important observations about the use of this test are as follows:

1. It may be used with a discrete or continuous distribution.

2. It allows the use of estimation of parameters and underlying distribution.

3. It requires "large" sample size (i.e., 20 or more).

The steps necessary to apply the Chi-square goodness-of-fit test are given below:

1. Partition the range of the variable into intervals with each interval closed from the left side.

2. Determine the number of sample observations falling within each interval.

3. Determine the level of significance, which is defined as the risk of rejecting the underlying distribution if it is, in fact, the real distribution.

4. Compute

$$\chi^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$$

where

O_i = Number of sample observations in the i^{th} interval

E_i = Expected number of observations in the i^{th} interval

N = Number of intervals

5. Determine if χ^2 is greater than $\chi^2_{\alpha,\ N-P-1}$

where

P is number of parameters estimated from the data.

N is number of interval lengths

α is risk level

and $\chi^2_{\alpha, N-P-1}$ may be found in a χ^2 table. If χ^2 is larger than $\chi^2_{\alpha, N-P-1}$, a decision is made to reject the distribution under test, otherwise, we do not have sufficient evidence to reject the assumed distribution.

The use of original repair times data without adjusting interval length is unrealistic since the recorders apparently rounded off these repair times. An analysis of original data with one-tenth hour intervals (Table II) was used to verify this conjecture. Subsequently, some of these interval lengths (Table II) were extended to include more sample observations in each of these intervals (Table III). The data show that the recorders tended to round the actual time it takes to repair an item to the nearest 12 minutes (.2 hour), 30 minutes (.50 hour), and 1 hour. Table II shows that out of 366 corrective repair maintenance times, there are 54, 148 and 67 of such occurring at .2, .5 and 1.0 hour, respectively. The selection of the interval widths for the Chi-square test was based on the rounding phenomena since the expected number of observations in any interval must be at least five for successful use of the Chi-square test, width 2.25 to 4.05 was selected. The risk level (α) of .05 using Chi-square test is generally used in rejecting the assumed distribution if the assumption is true. The χ^2 value for this distribution is

$$\chi^2_{\alpha, N-P-1} = \chi^2_{.05, 18}$$

$$\simeq 29$$

where

$\alpha = .05$

$N = 20$

$P = 1$

based on unadjusted data interval used in Table II. Also, the χ^2 value of

$$\chi^2_{\alpha, N-P-1} = \chi^2_{.05, 5}$$

$$\simeq 11.07$$

where

$\alpha = .05$

$N = 7$

$P = 1$

is based on the grouped data intervals in Table III. The goodness-of-fit test uses these tables Chi-square values of 29 and 11.07 to compare with the calculated $\sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$ of 977 (Table IV) and $\sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$ of 73 Table V, respectively. As a result, both χ^2 values are less than both table values which indicates that the assumed distribution is not exponentially distributed.

III. WEIBULL DISTRIBUTION

The Weibull distribution, considered in this section, is a possible model for repair times. Such a distribution is given by the following equations:

$$F(X) = \begin{cases} 1-\mathrm{EXP}\left(-\left(\frac{X}{\eta}\right)^{\hat{\beta}}\right) & X>0 \\ & \eta>0 \\ 0 \text{ otherwise} & \end{cases}$$

where

X = Maintenance repair time widths

η = The scale parameter

β = The shape parameter

Some of the same procedures are applied to this distribution as the exponential distribution as noted above. Applying the method of matching moments to estimate the parameters, η and β are calculated from the following respective equations using previously calculated estimations for mean θ and standard deviation σ.

$$\hat{\eta} = \frac{\hat{\theta}}{\Gamma\left(\frac{1}{\beta} + 1\right)} \text{ is the scale parameter}$$

where

$\hat{\beta} = 1/b$ is the shape parameter

or $b = 1/\hat{\beta}$

The b is calculated from equation

$$1 + \frac{\hat{\sigma}^2}{\hat{\theta}^2} = \frac{\Gamma\left(\frac{2}{\hat{\beta}} + 1\right)}{\left(\Gamma\left(\frac{1}{\hat{\beta}} + 1\right)\right)^2}$$

$$1 + \frac{\hat{\sigma}^2}{\hat{\theta}^2} = \frac{2\Gamma(2b)}{b\ (\Gamma(b))^2}$$

$$1 + \frac{(.6021)^2}{(.6634)^2} = \frac{2\Gamma(2b))}{b(\Gamma(b))^2}$$

$$1.82 = \frac{2\Gamma(2b))}{b(\Gamma(b))^2}$$

which must be solved first by iteration below and using a gamma function table.

b	$2\Gamma(2b))$	$b(\Gamma(b))^2$	$\frac{2\Gamma(2b)}{b(\Gamma(b))^2}$
1	$2\Gamma(2) = 2$	$1(\Gamma(1))^2=1$	2
.9	$2\Gamma(1.8) =$ 1.8628	$.9(\Gamma(.9))^2=$ 1.0278	1.82

Therefore b = .9 satisfies the above relationship, now an estimate for β is calculated below.

$$\hat{\beta} = \frac{1}{.9}$$

$$\hat{\beta} = 1.11$$

and the estimate for

$$\hat{\eta} = \frac{.6634}{\Gamma(1.9)}$$

$$= \frac{.6634}{.96177}$$

$$= .6898$$

The other change is the computation of the expected number of frequencies (E_i)

$$E_i = N\ (e^{-(\frac{X_i}{\eta})^{\hat{\beta}}} - e^{-(\frac{X_{i+1}}{\hat{\eta}})^{\hat{\beta}}})$$

where

$\hat{\beta}$ and $\hat{\eta}$ defined above

X_i = Intervals widths

N = Number of observations = 366

The χ^2 values for this distribution differ from the exponential distribution. The two parameter Weibull distribution is used to calculate $\chi^2_{\alpha,\ N-P-1}$ values for $\chi^2_{.05,17} = 28$ and $\chi^2_{.05,\ 4} = 9.5$ given in respective data tables II and III. Note that these values for

$$28 < \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i} = 876$$

and

$$9.5 < \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i} = 54$$

imply that the assumed distribution is not a Weibull distribution.

IV. GAMMA DISTRIBUTION

The gamma distribution is applied to these data to test how well such data fit this assumption. This analysis is similar to those presented earlier. A method for estimating the parameters β (scale parameter) and α (shape parameter) is the maximum likelihood procedure based on the gamma density function.

(1) $f(X;\beta,\alpha) = (\beta^{\alpha} X^{\alpha-1} \quad e^{-\beta X})/\Gamma(\alpha)$

the natural log of ℓ is L

where $\ell = \prod_{i=1}^{N} f(X;\beta,\alpha)$ $\qquad$ α is the shape parameter
β is the scale parameter

(2) $$L = \alpha N \ln \beta - N \ln (\Gamma(\alpha)) + (\alpha-1) \sum_{i=1}^{N} \ln (X_i) - \beta \sum_{i=1}^{N} X_i$$

(3) $$\frac{\partial L}{\partial \alpha} = N \ln \beta - N \phi(\alpha) + \sum_{i=1}^{N} \ln (X_i) = 0$$

$$(4)\quad \frac{\partial L}{\partial \beta} = \frac{\alpha N}{\beta} - \sum_{i=1}^{N} X_i = 0$$

Setting the partial derivatives in equations, 3 and 4 equal to 0.

where ϕ (α) is ln ($\Gamma(\alpha)$)

equations (3) and (4) can only be solved by trial and error, therefore equation (4) becomes

$$(5)\quad \hat{\alpha} = \frac{\beta}{N} \sum_{i=1}^{N} X_i = \hat{\beta}\, \bar{X}$$

where

N = Total Number of Samples

X_i = Individual Sample

$\bar{X}$ = Mean of Sample = .6634

β = Scale Parameter = 4.32186

α = Shape Parameter

Therefore α is estimated to be

α = 4.32186 (.6634) = 2.86712

and

$$(5)\quad \hat{\beta} = \exp\left(\phi(\hat{\alpha}) - \frac{1}{N} \sum_{i=1}^{N} \ln(x_i)\right)$$

which is simplified to

$$\hat{\beta} = k\, e^{\phi(\hat{\beta}\overline{X})}$$

where

$$k = e^{-\frac{1}{N} \sum_{i=1}^{N} \ln(x_i)}$$

The approximation for this is

$$e^{\phi(\hat{\beta}\overline{X})} \simeq (\hat{\beta}\,\overline{X} - \tfrac{1}{2})\left(1 + \frac{1}{24(\hat{\beta}X - 1/2)^2}\right)$$

$$e^{\phi(\beta\bar{x})} = (\hat{\beta}\overline{X} - 1/2) + \frac{1}{24(\hat{\beta}\overline{X} - 1/2)}$$

Therefore, the equation for β in (5) above resolves to a quadratic equation

$$a\hat{\beta}^2 + b\hat{\beta} + C = 0$$

Hence, the positive root of the above equation is

$$\hat{\beta} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Therefore

$$\hat{\beta} = 4.32186$$

These parameters may also be obtained by the method of matching moments, however it does not yield the minimum variance estimate for large samples, therefore, the maximum likelihood method was utilized. As a result, these estimated parameters and repair times are used in the underlying distribution analysis. Although the area under this distribution curve is not computable due to its non-closed form, it can be approximated using Simpson's Rule applied to the gamma density function (g.d.f.) that is

$$f(x) = \frac{\beta^{\alpha} X_i^{\alpha-1} e^{-\beta X_i}}{\Gamma(\alpha)} \qquad X>0$$

Tables VIII value of 605 and Table IX value of 80 give results of computerized g.d.f. calculations using original data intervals and adjusted data intervals respectively. The computer values of $\sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ in both tables above are compared to $\chi^2_{.05,\ 13}$ and $\chi^2_{.05,\ 3}$ for values of 605 and 80, respectively It is concluded that the Chi-square test result shows that this is not a gamma distribution.

V. TRUNCATED NORMAL DISTRIBUTION

The normal distribution is the best known distribution of statistics. It would certainly be appropriate to investigate maintenance times with the aid of the normal distribution. Unfortunately, this distribution has a characteristic which at times precludes its use with certain kinds of data. Essentially the normal distribution is defined for a random variable spanning the whole real line. The maintenance times now under study can never be negative and are best described by a random variable whose range is the positive half line. Still it is possible to use the normal distribution in the investigation of maintenance times if the forbidden region of the maintenance time random variable has a low probability of occurrence when described as the appropriate normal distribution. A rough rule of thumb to judge the applicability of the

normal distribution to positive data is to use this distribution only if the ratio of the sample mean to the sample standard deviation exceeds three. In our case this criterion is not satisfied. The mean of the sample data is .66 and the standard deviation estimate is .60 based on the following equation:

$$\theta = \sum_{i=1}^{n} \frac{x_i}{n} \text{ sample mean}$$

$$= \frac{242.8}{366}$$

$$= .6634$$

where x_i = each corrective repair time

n = Total number of observations

$n = 366$

and $$\hat{\sigma} = \sqrt{\frac{n \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}{n(n-1)}}$$

where x_i and n = same as above,

therefore $$\hat{\sigma} = \sqrt{\frac{107384.4 - 58951.84}{366(365)}}$$

$$\hat{\sigma} = \sqrt{\frac{48432.56}{133590}}$$

$$\hat{\sigma} = .6021$$

A normal variate with these values for mean and standard deviation would have a probability .14 of being negative. This is much too high a probability to be associated with data values which cannot possibly occur. In order to circumvent this difficulty the normal distribution may be truncated. In effect the area under the normal density curve to the left of some point, the truncation point, is ignored. The remaining part of the density function is multiplied by an appropriate constant to insure the total area under the curve is one. The truncated normal distribution is specified by either two or three parameters, depending on whether the truncation point is known. In the present instance the variable of interest is bounded by zero and thus there are only two parameters. If

μ = Mean Estimate and

σ = Standard Deviation Estimate of the untruncated normal distribution, then the density function f(x), of the truncated distribution is

$$f(x)=\begin{cases} \frac{1}{\sqrt{2\pi}\,\hat{\sigma}}\, e^{-1/2\,(\frac{x-\mu}{\sigma})^2} \cdot \frac{1}{1-G(x)}, & x>0 \\ 0 & x\leq 0 \end{cases}$$

where G is defined by

$$G(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} e^{-\frac{S^2}{2}}\, ds$$

and

$$1 - G(Z) = \frac{1}{\sqrt{2\pi}} \int_{Z}^{\infty} e^{-\frac{S^2}{2}}\, ds$$

$$S^2 = (\frac{x-\mu}{\sigma})^2$$

G(Z) is the cumulative probability function for the standard normal variate. It is convenient to define as an auxiliary parameter the standardized point of truncation TS. That is

$$TS = (\frac{x-\mu}{\sigma})$$

x = 0 for this case

therefore $T = -\frac{\mu}{\sigma}$

It should be clearly understood that even though three parameters will be estimated only two are needed to specify the distribution.

The method of matching moment will be used to estimate the parameters. According to Cohen, this procedure is equivalent to the maximum likelihood method, and in addition is easily described. Furthermore, along the way, expressions for the population mean and population standard deviation will be derived. The derivation begins with the computation of the first moment (M_1) and second moment (M_2) of the distribution.

$$M_1 = E(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \frac{1}{(1-G(TS))} \int_{TS}^{\infty} s e^{-1/2\,(\frac{s-\mu}{\sigma})^2}\, ds$$

$$M_2 = E(x^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \frac{1}{(1-G(TS))} \int_{TS}^{\infty} s^2 e^{-1/2\,(\frac{s-\mu}{\sigma})^2}\, ds$$

To simplify M_1:

Let $V = \frac{s-\mu}{\sigma}$

$$dv = \frac{ds}{\sigma}$$

Then

$$M_1 = \frac{1}{\sqrt{2\pi}} \frac{1}{[1-G(TS)]} \int_{TS}^{\infty} (\sigma V+\mu)\ e^{-\frac{V^2}{2}}\ dv$$

By integration

$$M_1 = \frac{1}{\sqrt{2\pi}} \frac{1}{[1-G(TS)]}\ \sigma\ e^{-\frac{(TS)^2}{2}} + \frac{\mu \int_{TS}^{\infty} e^{-\frac{V^2}{2}}\ dv}{[1-G(TS)]\ \sqrt{2\pi}}$$

As above let

$$G(Z) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{Z} e^{-\frac{S^2}{2}}\ ds$$

and define $L(x)$ by

$$L(x) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{[1-G(x)]} \cdot e^{-\frac{x^2}{2}}$$

By substitution of $L(x)$

$$M_1 = \sigma\ L(TS) + \mu = \text{Population Mean}$$

Now simplify M_2:

$$M_2 = \frac{1}{\sqrt{2\pi}} \frac{1}{[1-G(TS)]} \int_{TS}^{\infty} (\mu + \sigma V)^2\ e^{-\frac{V^2}{2}}\ dv$$

$$M_2 = \frac{1}{\sqrt{2\pi}} \frac{1}{[1-G(TS)]} \int_{TS}^{\infty} (\mu^2 + 2\sigma\mu\ V+\sigma^2 V^2)\ e^{-\frac{V^2}{2}}\ dv$$

By substitution of $G(TS)$

$$M_2 = \frac{\mu^2[1-G(TS)]}{[1-G(TS)]} + \frac{2(\sigma\mu) \int_{TS}^{\infty} V e^{-\frac{V^2}{2}}\ dv}{\sqrt{2\pi}\ [1-G(TS)]} + \sigma^2 \frac{\int_{TS}^{\infty} V^2\ e^{-\frac{V^2}{2}}\ dv}{\sqrt{2\pi}\ [1-G(TS)]}$$

By integration

$$M_2 = \mu^2 + \frac{2\,\sigma\mu\, e^{-\frac{(TS)^2}{2}}}{\sqrt{2\pi}\,[1-G(TS)]} + \frac{\sigma^2\, TS\, e^{-\frac{(TS)^2}{2}}}{\sqrt{2\pi}\,[1-G(TS)]} + \frac{\sigma^2 \int_{TS}^{\infty} e^{-\frac{V^2}{2}}}{\sqrt{2\pi}\,[1-G(TS)]}$$

Again by substitution of L(TS) and G(TS)

$$M_2 = \mu^2 + 2\,\sigma\mu L(TS) + \sigma^2\,(TS)\,L(TS) + \sigma^2$$

To obtain the estimate for μ, σ and TS the population first and second moments are replaced by the corresponding sample values. For this purpose the equations for μ, σ, and TS may be written as

$$\hat{M}_2 - \hat{M}_1{}^2 = \mu^2 + 2\sigma\mu\, L(TS) + \sigma^2(TS)L(TS) + \sigma^2 - (\sigma L(TS) + \mu)^2$$

$$\hat{M}_2 - \hat{M}_1{}^2 = \mu^2 + 2\sigma\mu\, L(TS)L(TS) + \sigma^2 - \sigma^2 L^2(TS) - 2\sigma\mu\, L(TS) - \mu^2$$

then

$$\hat{M}_2 - \hat{M}_1{}^2 = \sigma^2[TS\, L(TS) - L^2(TS) + 1]$$

Now

$$\hat{M}_1{}^2 = [\sigma\, L(TS) + \mu]^2$$

$$\hat{M}_1{}^2 = \sigma^2\, L^2(TS) + 2\sigma\mu L(TS) + \mu^2$$

Note that: $\hat{\mu} = -\,\hat{\sigma}TS = -1.7867\ (2.0173) = -3.6043$

Where $\hat{\mu}$ = associated normal mean and $\hat{\sigma}$ associated Normal Standard Deviation

$$\hat{M}_1{}^2 = \sigma^2\,[L^2(TS) - 2\,TS\,L(TS) + (TS)^2]$$

$$\frac{\hat{M}_2 - \hat{M}_1{}^2}{\hat{M}_1{}^2} = \frac{TS\ L\ (TS) - L^2\ (TS) + 1}{L^2\ (TS) - 2\ TS\ L\ (TS) + (TS)^2}$$

Since L(x) is not in closed form and an intergral part of equations $\hat{M}_1$ and $\hat{M}_2$, a computer program is used to calculate estimates for $\hat{M}_1$, $\hat{M}_2$, and for $\hat{\mu}$ and $\hat{\sigma}$, although $\hat{\mu}$ and $\hat{\sigma}$ are completed later. Now the ratio estimate for

$$\frac{\hat{M}_2-\hat{M}_1{}^2}{\hat{M}_1{}^2} = \frac{.8016-(.6634)}{(.6634)^2}$$

$$= \frac{.3615}{.4400996} = .8214$$

Where $\hat{M}_1 = .6634$

$\hat{M}_2 = .80164$

is computed.

Note: $\sigma = \dfrac{M_1}{L(TS)-TS}$

Since $TS = -\dfrac{\mu}{\sigma}$

$M_1 = \sigma\, L(TS) + \mu$

$M_1 = \sigma\, L(TS) - \sigma\, TS$

Therefore $\hat{\sigma} = \dfrac{M_1}{L(TS)-T(S)}$ which is

estimated to

$$\hat{\sigma} = \frac{.66339}{2.3886 - 2.0173}$$

$$\hat{\sigma} = \frac{.66339}{.3713}$$

$$\hat{\sigma} = 1.7867$$

These two parameters are now used in the truncated normal probability density function (P.d.f.) below to estimate the area under this assumed distribution curve.

$$\text{P.d.f.} \quad f(x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \frac{1}{[1-G(x)]} e^{-1/2 \frac{(x-\mu)^2}{\sigma^2}}$$

Since this distribution is not in closed form, it requires the use of a standard normal table to compute the area under the truncated normal distribution curve. Calculated results of such areas are generated in the computation of $\sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$ in Tables X and XI for unadjusted intervals and adjusted intervals, respectively. From Table X we obtain $\sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i} = 916$ which is greater than $\chi^2_{.05,\ 16} = 26$ and from Table XI $\sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i} = 68$ which is greater than $\chi^2_{.05,\ 4} = 9.5$.

The goodness of fit test indicates that the observed maintenance times do not come from a normal distribution.

VI. LOGNORMAL

The lognormal distribution requires that data be analyzed in similar manner as in previous sections. However, the natural log of individual maintenance repair times data from Table I is used to calculate the two parameters which are the sample mean ($\hat{\theta}_{ln}$) and standard deviation ($\hat{\sigma}_{ln}$).

1. $\hat{\theta}_{ln} = \frac{\sum \ln xi}{N} = \text{Sample mean}$

where

N = Total number of Maintenance Repair Times

$N = 366$

X_i = Individual Maintenance Repair Time

then

$$\hat{\theta}_{ln} = \frac{-257.55768}{366}$$

$$\hat{\theta}_{ln} = -.7037$$

and

2. $\hat{\sigma}_{ln} = \sqrt{\frac{N\sum(\ln X_i)^2 - (\sum \ln X_i)^2}{N(N-1)}}$ = Sample Standard Deviation

where

$N(N-1) = 366\ (365) = 133590$

$N\sum\ (\ln X_i)^2 = 145614.07$

$(\sum \ln X_i)^2 = 66335.958$

yields

$$\hat{\sigma}_{ln} = .7703$$

The lognormal distribution function is also not in closed form, however, its area under this curve can be approximated by similar methods as were used for the normal distribution approximation. First, a change of variable is made, that is from lognormal density function (l.d.f.).

$$f(x) = \frac{1}{X\sigma\sqrt{2\pi}}\ e^{-1/2\ \left(\frac{\ln x - \hat{\sigma}_{ln}}{\hat{\sigma}_{ln}}\right)}$$

let $y = \ln x$

$$\frac{dy}{dx} = \frac{1}{x}$$

Then

$$\frac{dx}{dy} = x$$

$f(x)$ transforms to $g(y) = f(x)\left|\frac{dx}{dy}\right|$

$$g(y) = \frac{1}{\hat{\sigma}\sqrt{2\pi}}\ e^{-1/2\ \left(\frac{y-\hat{\theta}}{\hat{\sigma}}\right)^2}$$ is standard normal density function

which is identical to normal density form.

Now, the area under the lognormal distribution can be found using the normal distribution. Finally, the Chi-square test at the same .05 risk level is used to determine if the assumed distribution is lognormal.

Tables XII and XIII show computerized results of $\sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}$

based on the same data intervals used in previous section. Table XII

value of 706 for $\sum_{i=1}^{18} \frac{(O_i - E_i)^2}{E_i}$ and Table XIII value of 12.79 are compared

with $\chi^2_{.05,\ 15}$ and $\chi^2_{.05,\ 4}$, respectively.

These comparisons contributed the results of

$\chi^2_{.05,\ 15}$ = 25 which is less than 706

$\chi^2_{.05,\ 4}$ = 9.49 which is less than 12.79 which infer that such lognormal distribution assumption is invalid at the .05 risk level. However, for this particular distribution another risk level (.01) can be used to satisfy the Chi-square test based on the adjusted intervals due to rationale stated in the introduction. Such information is shown by $\chi^2_{.01,\ 4}$ = 13.28 which is greater than 12.79. Therefore, this means that at the .01 risk level there is not sufficient evidence to say that this distribution is not lognormally distributed.

VII. CONCLUSIONS

The analytical results of each section are compared to determine if the discussed distribution can be used to model these data. Table XIV gives a summary of calculated data ratios which are compared to the χ^2 goodness-of-fit test at the 5 percent risk level. It is seen that each distribution's computed ratio yielded a larger value than its corresponding χ^2 value for unadjusted as well as adjusted intervals. However, the lognormal distribution does come closest to satisfying such tests. Therefore, based on this analysis the lognormal distribution is the best of these five models for the maintenance data analyzed.

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APPENDIX

TABLE I

Maintenance Repair Times (Hours)

.1	.5	.2	1.0	.5	.5	.5	3.0	.5	.2
4.0	.1	.1	.2	.5	.2	.5	.5	.5	1.0
.5	.5	.3	1.0	1.0	.2	.2	.5	.2	1.0
.5	.5	.5	.5	.2	.5	.5	1.0	.2	.3
.5	.3	1.0	1.0	.5	.5	1.0	.2	.5	.1
1.0	.5	.3	.3	.2	.2	.4	.5	.5	.2
1.0	.2	.5	.7	.1	.5	.2	.1	.5	.5
.5	.5	.5	.2	.5	.5	.6	.1	.5	.5
.4	.5	1.0	.5	.5	.5	.2	.6	1.0	.5
.2	.2	4.0	.6	.5	.5	.5	.5	2.0	1.5
.6	.5	1.0	.2	.1	.1	.1	1.0	.5	.5
.5	1.0	.5	.2	.5	.6	.3	3.5	.1	.2
1.5	.5	.5	.5	1.0	.5	.3	.3	2.0	.4
2.0	.2	.3	.5	.5	1.0	.5	4.0	.5	.2
.4	.5	1.3	.2	.5	.5	1.0	.5	.5	.5
1.0	.5	.5	.6	.4	.5	.2	.5	1.0	.5
.5	1.0	.5	2.5	1.0	.2	.5	1.5	.3	.5
.5	.5	.5	.5	.5	.5	.5	.5	.3	.5
1.0	.3	.5	.1	1.0	.5	1.0	.5	.5	.5
.2	.2	2.5	2.0	.7	.5	2.0	1.0	.5	1.0
.5	1.0	.2	.5	.5	1.0	.5	.1	1.0	.5
.1	.5	.5	.5	1.0	.5	.5	.2	.7	1.0

TABLE I (Continued)

Maintenance Repair Times (Hours)

.1	1.0	.2	1.5	.5	.5	.5	1.0	.5	.2
.5	2.0	1.0	.7	.7	1.0	1.0	.5	1.0	.5
.5	1.0	.2	.5	1.2	1.0	.6	.4	.1	.5
.5	.5	2.0	.2	1.0	.1	1.0	.9	.5	.1
.5	.5	1.5	.5	1.0	.2	1.0	.5	1.5	1.0
4.0	.5	.5	.5	.5	.5	2.0	.5	.5	.2
.5	.8	.5	.1	.1	.5	.5	1.0	1.0	.5
.2	.9	.5	.3	.5	.5	.2	.5	1.0	1.0
1.5	.2	.5	.2	.8	.2	1.0	.2	.2	1.0
.5	1.0	.5	.2	1.0	.1	3.0	.9	.5	.2
.2	1.0	.5	1.0	.5	.2	.2	.1	.9	1.2
1.0	.1	.6	1.0	1.0	.5	.5	.5	.2	.1
1.0	.6	.5	.1	.2	2.0	.5	.5	.5	.2
.2	1.0	1.0	.4	.1	1.0	.5	1.5	1.0	1.0
.5	.2	1.0	3.0	1.0	.4				

TABLE II

None Grouped Frequency Distribution of Repair Times

Class Intervals	Frequency	Cumulative Frequency
0 - .1	26	26
.1 - .2	54	80
.2 - .3	13	93
.3 - .4	8	101
.4 - .5	148	249
.5 - .6	9	258
.6 - .7	5	263
.7 - .8	2	265
.8 - .9	4	269
.9 - 1.0	67	336
1.0 - 1.1	0	336
1.1 - 1.2	2	338
1.2 - 1.3	1	339
1.3 - 1.4	0	339
1.4 - 1.5	8	347
1.5 - 1.6	0	347
1.6 - 1.7	0	347
1.7 - 1.8	0	347
1.8 - 1.9	0	347
1.9 - 2.0	9	356

TABLE II (Continued)

None Grouped Frequency Distribution of Repair Times

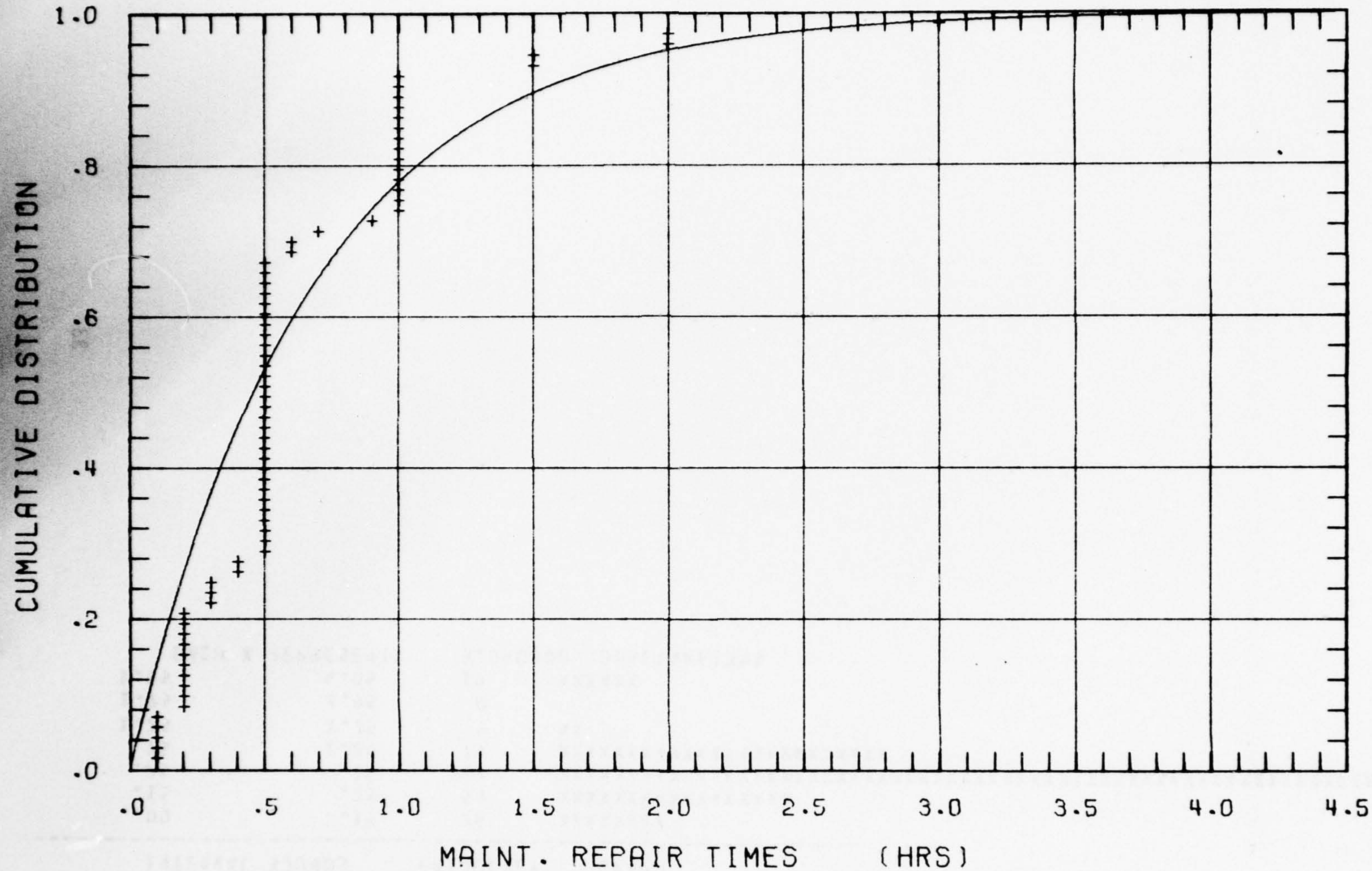
Class Intervals	Frequency	Cumulative Frequency
2.0 - 2.1	0	356
2.1 - 2.2	0	356
2.2 - 2.3	0	356
2.3 - 2.4	0	356
2.4 - 2.5	2	358
2.5 - 2.6	0	358
2.6 - 2.7	0	358
2.7 - 2.8	0	358
2.8 - 2.9	0	358
2.9 - 3.0	3	361
3.0 - 3.1	0	361
3.1 - 3.2	0	361
3.2 - 3.3	0	361
3.3 - 3.4	0	361
3.4 - 3.5	1	362
3.5 - 3.6	0	362
3.6 - 3.7	0	362
3.7 - 3.8	0	362
3.8 - 3.9	0	362
3.9 - 4.05	4	366

```
BELOW IS A BAR GRAPH OF THE DATA PARTITIONED INTO 40 INTERVALS

        INTERVAL BOUNDS      FREQUENCY    GRAPH
------------------------------------------------------------
      .00           .10          0
      .10           .20         26       XXXXXXXXXX
      .20           .30         54       XXXXXXXXXXXXXXXXXXXXXX
      .30           .40         13       XXXXX
      .40           .50          8       XXX
      .50           .60        148       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      .60           .70          9       XXX
      .70           .80          5       XX
      .80           .90          2
      .90          1.00          4       X
     1.00          1.10         57       XXXXXXXXXXXXXXXXXXXXXXX
     1.10          1.20          0
     1.20          1.30          2
     1.30          1.40          1
     1.40          1.50          0
     1.50          1.60          8       XXX
     1.60          1.70          0
     1.70          1.80          0
     1.80          1.90          0
     1.90          2.00          0
     2.00          2.10          9       XXX
     2.10          2.20          0
     2.20          2.30          0
     2.30          2.40          0
     2.40          2.50          0
     2.50          2.60          2
     2.60          2.70          0
     2.70          2.80          0
     2.80          2.90          0
     2.90          3.00          0
     3.00          3.10          3       X
     3.10          3.20          0
     3.20          3.30          0
     3.30          3.40          0
     3.40          3.50          0
     3.50          3.60          1
     3.60          3.70          0
     3.70          3.80          0
     3.80          3.90          0
     3.90          4.05          4       X
        EACH X REPRESENTS    2.466667  OBSERVATIONS
```

Figure II

```
BELOW IS A BAR GRAPH OF THE DATA PARTITIONED INTO  7 INTERVALS

      INTERVAL BOUNDS       FREQUENCY    GRAPH
-----------------------------------------------------------------
     .00           .15         26     XXXXXXXX
     .15           .25         54     XXXXXXXXXXXXXXXXX
     .25           .75        183     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
     .75          1.25         75     XXXXXXXXXXXXXXXXXXXXXXXX
    1.25          1.75          9     XX
    1.75          1.95          0
    1.95          4.05         19     XXXXXX
        EACH X REPRESENTS      3.050000  OBSERVATIONS
```


Figure II



Each 5th Observation is Represented by +

FIGURE III

TABLE III

Grouped Frequency Distribution of Repair Times

Class Intervals	Frequency	Cumulative Frequency
0 - .15	26	26
.15 - .25	54	80
.25 - .75	183	263
.75 - 1.25	75	338
1.25 - 1.75	9	347
1.75 - 2.25	9	356
2.25 - 4.05	10	366

EXPONENTIAL

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	74.07	31.19	31.19
.15	-	.25	54.00	40.85	4.23	35.43
.25	-	.35	13.00	35.13	13.94	49.37
.35	-	.45	8.00	30.22	16.34	65.71
.45	-	.55	148.00	25.99	572.80	638.51
.55	-	.65	9.00	22.35	7.98	646.49
.65	-	.75	5.00	19.22	10.53	657.01
.75	-	.85	2.00	16.53	12.78	669.79
.85	-	.95	4.00	14.22	7.35	677.13
.95	-	1.05	67.00	12.23	245.26	922.38
1.05	-	1.15	.00	10.52	10.52	932.90
1.15	-	1.25	2.00	9.05	5.49	938.39
1.25	-	1.35	1.00	7.78	5.91	944.30
1.35	-	1.45	.00	6.69	6.69	950.99
1.45	-	1.55	8.00	5.76	.87	951.87
1.55	-	1.75	.00	9.21	9.21	961.07
1.75	-	1.95	.00	6.81	6.81	967.89
1.95	-	2.15	9.00	5.04	3.11	971.00
2.15	-	2.45	.00	5.21	5.21	976.21
2.45	-	4.05	10.00	8.29	.35	976.56

Table IV

EXPONENTIAL

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	74.07	31.19	31.19
.15	-	.25	54.00	40.85	4.23	35.43
.25	-	.75	183.00	132.92	18.87	54.30
.75	-	1.25	75.00	62.55	2.48	56.77
1.25	-	1.75	9.00	29.44	14.19	70.97
1.75	-	2.25	9.00	13.86	1.70	72.67
2.25	-	4.05	10.00	11.50	.20	72.86

Table V

WEIBULL

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	61.47	20.47	20.47
.15	-	.25	54.00	39.86	5.02	25.48
.25	-	.35	13.00	36.13	14.81	40.29
.35	-	.45	8.00	32.13	18.12	58.41
.45	-	.55	148.00	28.25	507.58	565.99
.55	-	.65	9.00	24.64	9.93	575.92
.65	-	.75	5.00	21.36	12.53	588.45
.75	-	.85	2.00	18.43	14.65	603.10
.85	-	.95	4.00	15.84	8.85	611.95
.95	-	1.05	67.00	13.56	210.53	822.48
1.05	-	1.15	.00	11.58	11.58	834.06
1.15	-	1.25	2.00	9.86	6.27	840.33
1.25	-	1.35	1.00	8.38	6.50	846.83
1.35	-	1.45	.00	7.10	7.10	853.93
1.45	-	1.55	8.00	6.01	.66	854.59
1.55	-	1.65	.00	5.08	5.08	859.67
1.65	-	1.85	.00	7.89	7.89	867.56
1.85	-	2.05	9.00	5.58	2.10	869.65
2.05	-	2.35	.00	5.42	5.42	875.07
2.35	-	4.05	10.00	7.13	1.16	876.23

Table VI

WEIBULL

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	61.47	20.47	20.47
.15	-	.25	54.00	39.86	5.02	25.48
.25	-	.75	183.00	142.52	11.50	36.98
.75	-	1.25	75.00	69.28	.47	37.46
1.25	-	1.75	9.00	30.86	15.48	52.94
1.75	-	2.25	9.00	13.11	1.29	54.23
2.25	-	4.05	10.00	8.62	.22	54.45

Table VII

GAMMA

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	12.94	13.17	13.17
.15	-	.25	54.00	28.34	23.23	36.40
.25	-	.35	13.00	39.29	17.59	53.99
.35	-	.45	8.00	43.74	29.20	83.19
.45	-	.55	148.00	43.14	254.88	338.08
.55	-	.65	9.00	39.41	23.47	361.54
.65	-	.75	5.00	34.15	24.88	386.43
.75	-	.85	2.00	28.47	24.61	411.04
.85	-	.95	4.00	23.04	15.74	426.77
.95	-	1.05	67.00	18.22	130.63	557.40
1.05	-	1.15	.00	14.13	14.13	571.54
1.15	-	1.25	2.00	10.80	7.17	578.70
1.25	-	1.35	1.00	8.14	6.26	584.97
1.35	-	1.45	.00	6.07	6.07	591.04
1.45	-	1.55	8.00	7.77	.01	591.05
1.65	-	4.05	19.00	8.35	13.58	604.63

Table VIII

GAMMA

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	12.94	13.17	13.17
.15	-	.25	54.00	28.34	23.23	36.40
.25	-	.75	183.00	199.73	1.40	37.80
.75	-	1.25	75.00	94.66	4.08	41.89
1.25	-	1.75	9.00	24.37	9.69	51.58
1.75	-	4.05	19.00	5.96	28.51	80.08

Table IX

TRUNCATED NORMAL

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	67.44	25.46	25.46
.15	-	.25	54.00	38.31	5.94	31.41
.25	-	.35	13.00	34.40	13.31	44.72
.35	-	.45	8.00	30.39	16.50	61.22
.45	-	.55	148.00	26.77	549.03	610.25
.55	-	.65	9.00	23.50	8.95	619.20
.65	-	.75	5.00	20.57	11.79	630.98
.75	-	.85	2.00	17.95	14.17	645.15
.85	-	.95	4.00	15.61	8.64	653.79
.95	-	1.05	67.00	13.54	211.15	864.94
1.05	-	1.15	.00	11.70	11.70	876.64
1.15	-	1.25	2.00	10.08	6.48	883.12
1.25	-	1.35	1.00	8.66	6.78	889.90
1.35	-	1.45	.00	7.42	7.42	897.31
1.45	-	1.55	8.00	6.33	.44	897.75
1.55	-	1.65	.00	5.39	5.39	903.14
1.65	-	1.75	.00	4.57	4.57	907.71
1.75	-	1.95	.00	7.12	7.12	914.83
1.95	-	4.05	19.00	15.60	.74	915.57

TABLE X

TRUNCATED NORMAL

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	67.44	25.46	25.46
.15	-	.25	54.00	38.81	5.94	31.41
.25	-	.75	183.00	135.64	16.54	47.94
.75	-	1.25	75.00	68.86	.54	48.49
1.25	-	1.75	9.00	32.36	16.86	65.35
1.75	-	1.95	9.00	7.12	.50	65.85
1.95	-	4.05	10.00	15.60	2.01	67.86

TABLE XI

LOGNORMAL

INTERVAL		(HRS)	OBSERVED	EXPECTED	(O-E)**2/E	SUM
.00	-	.15	26.00	22.21	.65	.65
.15	-	.25	54.00	46.53	1.20	1.85
.25	-	.35	13.00	50.81	28.13	29.98
.35	-	.45	8.00	45.53	30.94	60.92
.45	-	.55	148.00	37.92	319.49	380.41
.55	-	.65	9.00	30.66	15.30	395.72
.65	-	.75	5.00	24.51	15.53	411.25
.75	-	.85	2.00	19.55	15.75	427.00
.85	-	.95	4.00	15.61	8.64	435.64
.95	-	1.05	67.00	12.52	237.19	672.82
1.05	-	1.15	.00	10.08	10.08	682.90
1.15	-	1.25	2.00	8.17	4.66	687.56
1.25	-	1.35	1.00	6.65	4.80	692.37
1.35	-	1.45	.00	5.45	5.45	697.82
1.45	-	1.55	8.00	8.21	.01	697.82
1.65	-	1.85	.00	5.68	5.68	703.51
1.85	-	2.15	9.00	5.56	2.13	705.63
2.15	-	4.05	10.00	9.18	.07	705.71

Table XII

LOGNORMAL

INTERVAL (HRS)			OBSERVED	EXPECTED	(O-E)**2/E	SUM
.0	-	.15	26.00	22.21	.65	.65
.15	-	.25	54.00	46.53	1.20	1.85
.25	-	.75	183.00	189.44	.22	2.07
.75	-	1.25	75.00	65.92	1.25	3.32
1.25	-	1.75	9.00	23.41	8.87	12.18
1.75	-	2.25	9.00	9.47	.02	12.21
2.25	-	4.05	10.00	7.86	.58	12.79

Table XIII

TABLE XIV

DISTRIBUTION	Unadjusted Intervals: $\chi^2_{\alpha, N-p-1}$ Table Value	Unadjusted Intervals: Computed Ratio of $\sum\frac{(O - E)^2}{E}$	Adjusted Intervals: $\chi^2_{\alpha, N-p-1}$ Table Value	Adjusted Intervals: Computed Ratio of $\sum\frac{(O - E)^2}{E}$
Exponential	$\chi^2_{.05, 18} \simeq 29$	977	$\chi^2_{.05, 5} = 11.1$	73
Weibull	$\chi^2_{.05, 17} \simeq 28$	876	$\chi^2_{.05, 4} = 9.5$	54
Gamma	$\chi^2_{.05, 13} \simeq 22$	604	$\chi^2_{.05, 3} = 7.8$	80
Truncated Normal	$\chi^2_{.05, 16} \simeq 26$	916	$\chi^2_{.05, 4} = 9.5$	68
Lognormal	$\chi^2_{.05, 15} \simeq 25$	706	$\chi^2_{.05, 4} = 9.5$	12.8

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